La Crema: A Case Study of Mutual Fire Insurance*

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December, 2000
Revised: February, 2001

Abstract

We analyze a mutual fire insurance mechanism used in Andorra, which is called *La Crema* in the local language. This mechanism relies on households' announced property values to determine how much a household is reimbursed in the case of a fire and how payments are apportioned among other households. The only Pareto efficient allocation reachable through the mechanism requires that all households honestly report the true value of their property. However, such honest reporting is not an equilibrium except in the extreme case where the property values are identical for all households. Nevertheless, as the size of the society becomes large, the benefits from deviating from truthful reporting vanish, and all of the non-degenerate equilibria of the mechanism are nearly truthful and approximately Pareto efficient.

Keywords: insurance, contract theory, mechanism design, truthful revelation.

JEL Classification: A13, C72, D64, D80.

*We are grateful to Francisco Alcalá, Luis Corchón, Ashok Rai, Rafael Repullo, Joel Sobel and the seminar participants at the European Winter Workshop of the Econometric Society and the UCI - Development Economics Conference for their comments. We also gratefully acknowledge the financial support of Spain's Ministry of Education under grant PB96-0302 and the Generalitat de Catalunya under grant 1999SGR-00157 and the National Science Foundation under grant SES-9986190.

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1 Introduction

Mutual insurance companies write large proportions of insurance policies in many sectors.[1] They have been very successful for several reasons. First, as Malinvaud (1973) points out, future markets provide only a remote idealization to the actual mechanism for risk allocation since "the ideal market system is too costly to implement." On the contrary, pooling individual risk by means of mutual insurance policies "permits substantial economizing on market transactions" (Cass, Chichilnisky and Wu, 1996). Another important reason for the success of mutual insurance is that they can solve through peer monitoring some moral hazard problems that plague incorporated insurance companies.[2] [3] While these problems are well understood, mutual insurance arrangements also solve other informational problems relating to the discovery of the value of insured property, as we show here.

In this paper we present and analyze a real-life mutual fire insurance mechanism that has been functioning in a rural mountainous area of Western Europe for well over a century and a half. In this mechanism, called *La Crema* in the local language, each participating household must report a *value*. In case there is a fire, the owner of the burned household receives her reported value, which is paid by all participating households (including herself) in proportion to their reported values. We focus on the rules of *La Crema* because they are particularly clear from a game-theoretic point of view, they are by no means exceptional, and the mechanism has some remarkable properties.[4]

In particular, the properties of the *La Crema* mechanism that we explore concern its efficiency characteristics and the incentives it provides for truthful reporting of property values. One important characteristic is that the

[1] "Advance premium mutuals write almost 40 percent of the life insurance in force and almost 23 percent of the property and liability insurance premiums." (Williams, Smith and Young, 1998).

[2] "Mutuals seem to have been more effective than stock companies in constructing such incentive systems, particularly in the early phases of their history. Individual industrialists were sometimes large enough to make investment in research on fire prevention worthwhile, but stock companies discouraged the provision of public goods by appropriating too much of the saving from decreased fire losses" (Heiner, 1985).

[3] Obviously, mutuals have problems of their own, or they would be the only organizational form. "From a financial perspective, the key impediment to mutual life company stability, growth and development, is that equity capital can be raised only through retained earnings from the company's operations," (Garber, 1993). Also, mutuals are very difficult to take over, which makes the corporate governance problem harder to solve, especially in large mutuals.

[4] A similar proportion rule is adopted, for instance, in marine insurance clubs: "At the beginning of the year the shipowners are given an estimate of the amount (call) they will be required to pay into the [Protection and Indemnity] Club. However, the eventual call is dependent upon the claim made by all members: each member knows only the *proportion* [emphasis in the original] of the total cost they will be required to bear." (Bennett, 2000).

mechanism allows for announcement of any value by households and does not seek any appraisal or cross-report by any witnesses. This is potentially a nice feature because it could allow the mechanism to insure the "subjective" value of property (as a welfarist would like), rather than the appraisable market value. The subjective value can include sentimental factors which could not be valued appropriately by the market. This additional feature of the mechanism will only be useful if the mechanism provides incentives to (approximately) announce truthfully and provides for efficient risk sharing. We will see that, under appropriate conditions, the mechanism performs these tasks quite well, and without having to resort to audits or other forms of "independent" assessments. Let us now discuss the mechanism's performance in more detail.

With regards to efficiency, the mechanism places strong constraints on the possible risk sharing that can take place since reimbursements and payments are both scaled directly in terms of the announced property values. For instance, if households have constant (and identical) relative risk aversion, the only Pareto efficient allocation that is reachable through the game requires that all households truthfully report the value of their property. Things are even worse with constant (and identical) absolute risk aversion as then no Pareto efficient allocation is obtainable as an outcome in the game regardless of how the announcements are varied.[5]

With regards to the incentives that the mechanism provides for truthful reporting of property values, we show that there is an equilibrium where all households report the true value of their insured property if and only if these valuations are exactly the same across households. Apart from this extreme case of identical property values, we show that households with relatively high property values have an incentive to overreport their value (to increase reimbursement from others when needed) and households with low property values have an incentive to underreport their value (to decrease payment to others when asked for).

The analysis described above appears to be in conflict with the conventional wisdom among the actual participants in the game, who are happy with the functioning of the mechanism and consider that the only natural thing one can do is to report the true value of the property. Since the mechanism has existed for a long time one would think that tradition or their own experience could furnish enough information for agents to know their best response. In fact, the incentive and efficiency properties that the mechanism exhibits are quite appealing and closely in line with local wisdom once we examine large enough societies and consider approximate rather than exact efficiency.

From the perspective of larger societies, we first show that households

[5]As one would expect, by the nature of the mechanism, where only property values are reported, differences in risk aversion do not seem to be the answer either.

in large enough societies have arbitrarily small incentives to deviate from honest reporting, or in other words, truth is an ε-Nash equilibrium. Second, we show that in large enough societies, the (exact) Nash equilibria of the *La Crema* mechanism involve reports that are arbitrarily close to the truth.[6] Third, the Nash equilibria (and ε-Nash equilibria) are arbitrarily close to being Pareto efficient in large enough societies. Finally, we show that for reasonable parameterizations of utility functions what is needed in the above statements in terms of "large enough" societies, can actually be reasonably small. Moreover, these results are robust to variations in the informational structure as they hold both with complete and with private information.

The interest of this institution is manifold, and quite different from other studies of risk sharing institutions.[7] First of all, the *La Crema* institution refers to a specialized type of risk, which limits the potential explanations for observed behavior. Secondly, the transfer rules are quite explicit and regulated. Finally, the rural society under consideration is relatively rich during the whole period of the mechanism's operation (for example, there are no instances of famines during its existence). The remainder of the paper proceeds as follows. Section 2 describes the mechanism (informally and formally) and gives some background on the society where the institution operates. Section 3 discusses the equilibrium and efficiency properties of the mechanism. Section 4 provides results characterizing the equilibria and approximate efficiency of the mechanism in "large" societies. Section 5 concludes.

2 *La Crema*

2.1 The institution of *La Crema*

In 1882, and under the initiative of the local priest, the 102 farms of Canillo in Principality of Andorra[8] organized themselves into a fire insurance cooper-

[6] One should note, of course, that while the larger scale of society may solve the reporting problem of *La Crema,* it may create other problems, as providing adequate fire prevention can become now a worse public good problem. "Today's P&I [Protection and Indemnity] Clubs are global in scale, with the largest containing over 20% of the world's oceangoing fleet. Communal responsibility may be unrealistic in such large-scale institutions because free rider problems become more difficult to monitor and control as group size and dispersion increase." (Bennett, 2000).

[7] Such as the ones mentioned in McCloskey (1989), Townsend (1993) or Fafchamps (1999). Besley, Coate and Loury (1993, 1994) examine the allocative performance of a simple, easily organized and widely observed institution for financial intermediation called *rosca* (rotating savings and credit associations).

[8] The Principality of Andorra, located in the heart of the Pyrenees between France and Spain, is both one of the smallest and the oldest states in Western Europe: the national territory is 468 km^2 and today's frontiers were definitely settled in 1278. The country is divided administratively into seven parishes: Canillo, Ordino, La Massana, Encamp, Andorra la Vella, Sant Julià de Lòria and Escaldes-Engordany. Agriculture has been the

ative named *La Crema*. By that time, Andorra was mostly a rural area living in quasi-autarchy, and *La Crema* was conceived as a risk-sharing institution to cope with fire damages that were a source of major worries to farmers in mountainous Canillo where sinuous and steep roads did not allow for quick nor effective fire brigades. Since its early beginnings, the role of *La Crema* was twofold: as a logistic structure, to organize the local fireman forces; as a financial structure, to guarantee pecuniary compensations to farms suffering fire destructions.[9]

The organization of *La Crema* is as follows. Once a year, the cooperative members meet in a general assembly, the *consell de La Crema* (*La Crema* council). The meeting is fixed on the Sunday that falls two weeks before the carnival and attendance is compulsory for all members.[10] The meeting is supervised by two permanent *secretaris* (secretaries) who are elected for life. During this general assembly, each farmer announces a value for each of the building that he or she owns (farm, barn, cow-shed, stable, etc.). Conventional wisdom suggests that farmers report the true and total value of their property, and *La Crema* cooperative members typically do so. This amount is noted in three different books: each *secretari* keeps a copy at home and a third book is stored at the parish town-hall.[11] In the case of a fire, the owner of the damaged building receives a compensation equal at most to the value noted in the book for the current year, depending on the extent of the damages. This financial compensation is made by the other cooperative members, who pay in proportion to the share their own announced property value represents with respect to the total of all values announced by the *La Crema* members. An early reference and brief description of the *La Crema* transfer rule can also be found in Brutails (1904): "Comme dans toute les populations aux prises avec une nature ingrate, la solidarité est développée parmi les Andorrans; elle a donné naissance à des sociétés d'assurances mutuelles contre l'incendie. Les sociétés d'assurances sont généralement ouvertes aux habitants d'un village; les associés peuvent refuser d'admettre au bénéfice de l'assurance les immeubles dont les risques dépassent la moyenne. En cas de sinistre, chacun paie, pour indemniser le propriétaire, au prorata de la somme pour laquelle lui-même est assuré" (p. 42).[12]

major economic activity of Andorra until the end of the 19^{th} century; tourism, commerce and financial services are now the basic national economic activities. In 1999, the GDP per capita was 20,252 $. See http://www.turisme.ad/angles/index.htm for more details.

[9] *La Crema* is still active and intervened recently to financially compensate Cal Soldevila whose barn partially burned in August of 1998 and Cal Batista for similar damages in July of 1985.

[10] An absent member without a good excuse is fined. The last fine dates back to 1946.

[11] We are indebted to one *secretari*, Josep Torres Babot (Cal Jep), and to the Canillo public librarian M^a Dolors Calvó Casal (Cal Soldevila) for their invaluable help in providing thorough information about *La Crema* during long conversations.

[12] "As it is often the case with societies living in inhospitable areas, solidarity is highly

During the yearly meeting, four *comissionats* (commissioners) and three *recaudadors* (money-collectors) are elected for one year. The *comissionats* are responsible for the logistic and technical activities. First, they guarantee that all cooperative members take the appropriate precautionary measures to prevent possible fires by reporting to the *consell de La Crema* carelessness in farm and building maintenance and to report any problematic behavior. Second, they are in charge of the fire fighting material owned by the cooperative (fire-hoses, etc.). Finally, in case of fire, the *comissionats* fix, in accordance with the concerned farmer, the total value of the damages to be reimbursed (depending on the extent of the damages and not exceeding the value noted in the book) and submit it to the *consell* for approval. The three elected *recaudadors* represent each a different geographical area: Canillo, la Ribera and Prats.[13] In case of fire, and once the amount to be transferred to the damaged farm is fixed by the *consell* under proposition of the *comissionats*, the *recaudadors* are responsible for collecting the contributions of the *La Crema* members within their area of intervention.

In the formal game theoretic analysis we are going to focus on the incentives to report truthfully the value of the property. As we mention in the introduction, the relevant valuation here is the individual subjective value, which may be very different from the market valuation. Because of this, there is quite a lot of freedom in the mechanism for reporting valuations. Unfortunately, this implies that there may be incentives to over-insure your property and then burn it. If players can commit arson (and not be caught), that would completely destroy any possibilities for any insurance (*La Crema* or otherwise), which is why commercial firms typically disallow insuring a property above its market price. Deterrents to arson are twofold: as for other insurance arrangements, there is a chance of being caught and suffer severe penalties (long prison terms). But *La Crema*, as other mutual insurance arrangements, also adds another dimension that a commercial or market based insurance scheme would not: given that each household is insured by their neighbors, the neighbors have an added incentive to monitor the behavior of a given household to make sure that they abide by the fire codes (and do not commit arson!).

developed among the Andorrans, and has given rise in particular to mutual fire-insurance associations. Inhabitants of a same village can usually all become insurance society fellows. Nonetheless, buildings offering fire-risks above average may be denied insurance coverage. In case of damage, all fellows pay to compensate the owner for her loss, and they do so in proportion to the value for which they are themselves insured."

[13]The first region, Canillo, corresponds to the main town with the same name. The second region, la Ribera, includes the following villages: Els Plans, Els Vilars, El Tarter, L'Aldosa, L'Armiana, Ransol and Soldeu. Finally, the last region, Prats, includes: El Forn, Meritxell, Molleres and Prats.

2.2 The *La Crema* game

There is a set N of households, with $|N| = n$. Each household has a utility function u_i and a wealth $w_i \in [c, C]$ where $C \geq c > 0$. Let $W = \sum_{i \in N} w_i$.[14] We take each u_i to be twice continuously differentiable and strictly concave.

Let $S = 2^N$ be the set of possible states. In particular, $s \in S$ is a list of farms that burned. For instance, $s = \{2, 7, 12\}$ denotes that farms 2, 7 and 12 (and only those farms) burned. Let $S^{(k)} = \{s \mid \#s = k\}$ be the set of states where exactly k farms burn. Note that $S = \cup_{k=0}^{n} S^{(k)}$. For any $i \in N$, let S_i denote the set of states for which farm i burns (perhaps along with some other farms), and $S_i^{(k)}$ be the set of states for which k farms in addition to farm i burn. Let p_s be the probability of state s. We assume that all states where an identical number k of farms burn are equally likely. That is, for all $s, s' \in S^{(k)}$, $p_s = p_{s'}$ and we denote this probability by p_k.[15] A special case of this is where each farm burns with an independent and identical probability. Note, however, that it is *not* required that the burnings be independent. As an extreme example, it could be that $p_0 > 0$ and $p_n > 0$ and $p_k = 0$ for all other k. This might be an example where all the farms lie close to each other in a forest, so that either all farms burn or none burns. All we assume is that $p_k > 0$ for some $k > 0$, so that there is some chance of a fire.

We now describe formally the rules of the *La Crema* game. Each household sends a message $m_i \in [0, 2C]$, to the coordinator, which is interpreted to be an announcement of their (subjective) property value at risk.[16] Let $m = (m_1, \ldots, m_n) \in [0, 2C]^n$ be a vector of messages. Let $M = \sum_{i \in N} m_i$ and for all $s \in S$, let $M_s = \sum_{i \in N \setminus s} m_i$. The allocation rule used by the coordinator is the following: in state $s \in S$, household $i \in s$ receives $m_i \frac{M_s}{M}$, whereas each household $j \in N \setminus s$ receives $w_j - m_j \frac{(M - M_s)}{M}$. One can easily check that $\sum_{j \in N \setminus s} m_j \frac{(M - M_s)}{M} = \sum_{i \in s} m_i \frac{M_s}{M}$, namely that the sum of the contributions by households $j \in N \setminus s$ whose farms did not burn is equal to the sum that households $i \in s$ receive as a compensation for their losses. Note that if announcements are truthful ($m_i = w_i$), then in each state s the undamaged property is effectively distributed among all households in proportion to their wealths (so the final allocations are $W_s \frac{w_i}{W}$).

[14] We treat wealth as the property that may potentially burn. Utility functions may, of course, be normalized so that this is without loss of generality.

[15] This condition is important in the approximate efficiency and equilibrium results we obtain. If this condition does not hold, so that there are some asymmetries in relative probabilities of different farms burning, then one could form sub-groups for insurance, where farms with similar probabilities were grouped together. This will become clear in the proofs of the propositions and in some discussion below.

[16] The upper bound on announcements is arbitrarily set at twice the highest imaginable property value. Any upper bound would do.

3 Discussion of the game

3.1 Equilibria

The first proposition says that truthful announcements are a Nash equilibrium only in the case where all wealths are identical.

Proposition 1 *The La Crema game has a Nash equilibrium in pure strategies where $m_i = w_i$ for all $i \in N$ if and only if $w_i = w_j$, $\forall i, j \in N$.*

The proof of Proposition 1 appears in the appendix. The intuition behind the Proposition is roughly as follows. Increasing m_i has two effects. First, it increases the reimbursement that household i receives in the case of a fire that consumes i's property. Second, it increases the liability that i faces in the event that some other household's property burns. Some heuristic calculations help illustrate the relative size of these two effects and the incentives that households have as a result. For simplicity, consider a situation where at most one household will have a fire, and so we need only consider states of the form $\{i\}$, where i's property is destroyed.[17] Consider what happens if i raises m_i by some small amount $\varepsilon > 0$. This increases i's reimbursement by (approximately) $\varepsilon \frac{M_i}{M}$ if the state is $\{i\}$ (where recall that $M_i = \sum_{j \neq i} m_j$ and $M = \sum_j m_j$). It also increases the payments that i has to make to household $j \neq i$ in state $\{j\}$ by $m_j \frac{\varepsilon}{M}$. Note that summing across states, these cancel each other out. That is, $\varepsilon \frac{M_i}{M} = \sum_{j \neq i} m_j \frac{\varepsilon}{M}$. So, by lowering the announcement m_i, household i transfers wealth from state $\{i\}$ to the other states $\{j\}$, $j \neq i$; and vice versa from raising the announcement. So what are the households' incentives in the game? Given their risk aversion, they wish to come as close as possible to smoothing their wealth across the states. If all households have exactly the same wealth, then at a truthful announcement in the *La Crema* game household i gets final wealth $w_i \frac{W_s}{W}$ in state s, and given the equal starting wealths is equal across each state $s = \{k\}$. Thus, the households' wealths are evenly spread across these states and they have no incentives to change their announcements. Next, consider the case where households do not have the same wealth. Order them so that $w_n \geq w_{n-1} \cdots w_1$, and $w_n > w_1$. Then notice that farmer 1 consumes the highest amount in the state where her property burns $w_1 \frac{W_1}{W}$, versus $w_1 \frac{W_j}{W}$ in some state $j \neq 1$, since $W_1 \geq W_2 \cdots \geq W_n$. By lowering m_1 a little, household 1 decreases consumption in the state $\{1\}$ where farm 1 burns, and distributes a commensurate increase among other states $\{j\}$,

[17] The state where no farm burns has no impact since no payments are made. States where several farms burn have analogous calculations as those discussed here, as the consideration is what happens if i's farm burns versus some other farm burns (on the margin).

where farm $j \neq 1$ burns. As households are risk averse, this strictly benefits household 1. Conversely, farmer n consumes less in the state where farm n burns compared to states where some other farm burns. By raising m_n, farmer n shifts wealth from states $\{j\}$, $j \neq n$, to state $\{n\}$. Roughly, households with below average property value will benefit from underreporting, and those with above average property value will benefit from overreporting.

The proposition tells us that the game does not have an equilibrium where households report the true value of their property if there is any heterogeneity in household value. The case of heterogeneity is arguably the interesting case, as it would be hard to see the reason for an elaborate mechanism (which is not costless to administer) unless there were some kind of heterogeneity. Otherwise, there would be common knowledge precisely about the thing that the coordinator is trying to elucidate.

This result still holds when there is private information about property values. All that is needed (this is clear from the proof as well as in the intuition above) is for some households to be fairly sure that they have the top or bottom property value (or that they are close to either).

The following remark shows that the problem goes even further. When there are only two households, there is no interior pure-strategy equilibrium to the game at all. Either both households refuse to participate (there is always such a degenerate equilibrium where neither household declares any wealth given the expectation that the other will not), or the wealthier household has such a strong incentive to overreport that they report the maximum allowed property value.

Remark 1 *Let $n = 2$. If $w_1\left(1 + \frac{w_2}{4C}\right) < w_2$ (a sufficient condition for which is $w_1 < \frac{3}{4}w_2$), then the only pure-strategy Nash equilibria of the La Crema game are $(m_1, m_2) = (0, 0)$ and $(m_1, m_2) = (\frac{2w_1C}{4C+w_1}, 2C)$.*

It is hard to see what an insurance mechanism is trying to accomplish if it leads to such extreme outcomes.

Before providing an answer to this paradox, let us examine the Pareto efficiency characteristics of the *La Crema* game.

3.2 Efficiency

Let $W_s = \sum_{i \in N} w_i - \sum_{i \in s} w_i$. Thus, W_s is the total wealth in the society given that s is the state. Let a *risk-sharing allocation* be any random vector $x = (x_1, \ldots, x_n)$ such that $\sum_{i \in N} x_i(s) = W_s$ in each state s. Thus, a risk-sharing allocation is some distribution of the wealth in the society. Note that this includes risk-sharing schemes that are not available as outcomes of the *La Crema* game. Let $Eu_i(x)$ denote the expected utility of $i \in N$ under the risk-sharing allocation x. Let x^m denote the risk-sharing allocation that comes from announcements m in the *La Crema* game. And let x^w denote the risk-sharing allocation that comes from truthful announcements ($m_i = w_i$) in the *La Crema* game.

We begin with efficiency results for the special case where households have identical constant relative risk aversion (CRRA) utility functions (i.e., $u_i(c_i) = c_i^\gamma/\gamma$ with $\gamma \neq 1$). We show that even in this special case the only Pareto efficient[18] allocations that can be reached as outcomes of the *La Crema* game arise from reporting the true value of one's household. The reason is that equality of marginal rates of substitution across states of the world requires that ratios of consumption are equalized for all states of the world. This can only happen when households report the true value of the property.

Proposition 2 *If households have identical CRRA utility functions and there exist $i, j \in N$ such that $w_i \neq w_j$, then there is a unique Pareto efficient risk-sharing allocation that is reachable through the La Crema game. It is to have each household report truthfully (so $x_i^w(s) = w_i \frac{W_s}{W}, \forall i \in N, \forall s \in S$).*

We note that Propositions 1 and 2 imply that the only Pareto efficient outcome of the *La Crema* game (under identical constant relative risk aversion) cannot be sustained as a Nash equilibrium.

Given that (Arrow-Debreu complete market) Walrasian outcomes are efficient, an interesting question in this context is whether the unique Pareto efficient outcome reachable through the *La Crema* game (when households have identical CRRA utility functions) corresponds to the Arrow- Debreu complete market Walrasian equilibrium of this economy when the endowments for the household i are w_i in state $s \notin S_i$ and 0 in states S_i. The following proposition shows that this is generically not the case.

Remark 2 *Let the probability of any farm burning be given by $p > 0$ and have this probability be independent across farms. If there exist k and j such that $w_k \neq w_j$, then the unique Pareto efficient allocation reachable through the La Crema game when the players have identical CRRA utility functions, is different from the outcome of the complete market Walrasian equilibrium of the La Crema economy.*

The next proposition shows that if agents have CARA utility functions, then difficulties in reaching efficiency are even worse for the *La Crema* game in that all of the allocations that are reachable through the game are inefficient. The reason is that Pareto optimality with identical CARA utility functions

[18]Pareto efficiency is, of course, relative to the expected utilities for an allocation. So expectations are taken before the state is realized and so households do not know which property has been destroyed.

requires that differences in utilities across states of the world are equalized across agents. This demands on the one hand that reports are the same for all agents, and at the same time that they are truthful. With heterogeneous endowments the two requirements are not compatible.

Proposition 3 *If for some $i, j \in N$, $w_i \neq w_j$ and households have identical CARA utility functions, then there is no Pareto efficient allocation that can be reached through the La Crema game.*

The following remark shows that differences in risk attitudes across households will not help to explain the inefficiency of the *La Crema* game. This is evident when the probability of no property burning is different from zero ($p_0 > 0$), because in that case Pareto efficiency requires transfers from the relatively more risk averse agents to the relatively less risk averse agents when no property burns (i.e., in state $s \in S^{(0)}$), and *La Crema* specifies no transfers for $s \in S^{(0)}$. The remark shows that even if there were some household burning in all states of the world ($p_0 = 0$), there would still be no Pareto efficient outcome of the game.

Remark 3 *Assume that $n \geq 3$, that household $i = 1$ is risk neutral, the other households have (possibly heterogeneous) CRRA utility functions, and for some $i, j \in N$, $w_i \neq w_j$, then there is no Pareto efficient allocation that is obtainable through the La Crema game.*

The above results leave us with a puzzle that needs to be explained. Pareto efficiency can only be obtained through the *La Crema* game in some extreme cases, and even then the corresponding allocation cannot be sustained as an equilibrium of this game as long as there is any heterogeneity in household property values. So why would the *La Crema* game be used? An analysis of larger societies provides an answer.

4 Larger Societies

While Proposition 1 shows that truth is only a Nash equilibrium in extreme (and implausible) situations, the *La Crema* game still has very nice features in terms of its equilibrium structure and efficiency characteristics. We point these out in a series of propositions. First, we show that truth is an ε-Nash equilibrium for large enough societies. Thus, the gains from over or understating one's wealth are not large. While this suggests that the *La Crema* game will have nice properties, it is not completely convincing since it does not guarantee that the exact Nash equilibria will be close to truthful. Second, we show that there always exist (non-degenerate) Nash equilibria. Third, we show that all non-degenerate Nash equilibria are close to truthful in large societies. Thus, the *La Crema* game provides incentives for individuals to play (approximately) truthfully. Finally, we show that truth and all announcements close to truth are approximately Pareto efficient (with arbitrary utility functions). Taken together these results show that the Nash equilibria and ε-Nash equilibria of the *La Crema* game are approximately efficient in large societies with arbitrary heterogeneity in preferences and endowments.

In order to talk about large societies and approximation, we consider the following setting. Let $n^1, n^2, n^3, \ldots$ an increasing sequence of integers such that $n^h \to \infty$. Each $h \in \mathbb{N}$ defines a *La Crema* game with population N^h of size n^h.

In addition, we maintain the following assumption on preferences in what follows. For all $i \in N^h$ and for all $h \in \mathbb{N}$:

($A1$) For any $\mu > 0$ there exists $\delta > 0$ such that if $|w - w_i| < \delta$ then $|u_i'(w) - u_i'(w_i)| < \mu$.

($A1$) implies that the second derivative of utility functions has some bound that applies to all players and games.[19] In other words, players are not arbitrarily risk averse. Note that no particular form is assumed for the utility functions u_i – so they can differ across people as long as there is an upper bound on how risk averse people are.

4.1 Approximate Equilibria

Proposition 4 *For any $\varepsilon > 0$ there exists an integer H such that for any $h > H$, it is an $\varepsilon-$Nash equilibrium of the La Crema game for all people in N^h to report truthfully ($m_i = w_i$).*

The proof of Proposition 4 appears in the appendix. To get a feeling for the intuition, let us do the following exercise. Changes of a given m_i have relatively little impact on $M = \sum_i m_i$ in a large society, so let us treat M as fixed – as the effects on it are second order (these effects are carefully handled in the appendix). Consider a scenario where one farm burns, but we are not sure which. So, the conditional expectation is $1/n$ on each farm. What happens if household i increases m_i by one unit? The gain is roughly $\frac{1}{n}\sum_{j \neq i} \frac{m_j}{M} u_i'(m_i - m_i^2/M)$, in the case where it is i's farm that burns. The loss is $\frac{1}{n}\sum_{j \neq i} \frac{1}{M} m_j u_i'(w_i - m_i m_j/M)$ as we sum over the cases where each other farm burns – as i is liable for an extra $1/M$ of each value m_j. Since in a large society $m_i/M \approx 0$, these approximately cancel at $m_i = w_i$, and so i does not gain much by changing m_i. So, under the *La Crema* game, the

[19]Note that this assumption trivially holds in the CRRA case as long as the γ is bounded from above.

expected cost (in utils) of the insurance is approximately $\sum_{j \neq i} m_j / M u_i'(w_i)$, and it pays off approximately $\sum_{j \neq i} m_j / M u_i'(m_i)$.[20]

Another way to view this, is to go back to the intuition discussed after Proposition 1. Lowering household i's announcement effectively transfers wealth from states where i's property burns to states where some other property burns in i's place. The relative difference in i's wealth across these states under truthful reporting is negligible to begin with: $w_i \frac{W_s}{W}$ is almost the same as $w_i \frac{W_{s'}}{W}$, if s is a state where i burns and s' is a corresponding state where some other farm burns in i's place; as $\frac{W_s}{W}$ is almost the same as $\frac{W_{s'}}{W}$ in a large society.

The above intuition shows that *La Crema* is a subtle institution since the cost of insurance depends on $u_i'(w_i)$ and its payoff depends on $u_i'(m_i)$ − and most importantly in a way that gives agents just the right incentives (in large economies where M is approximately unaffected by i's announcement).

Let us stress an important feature of the result in Proposition 4. The bounds we use in the proof are robust to the information structure and the actions of the other agents. That is, they do not depend on the p_k's, what the w_j's are for $j \neq i$, and work uniformly across i's so long as (A1) is satisfied.[21] In fact, all that is needed is that a household believes that their property value will be a relatively small amount of the total announced property value to have truth be nearly a best response. This robustness is important not just for realism's sake. In an environment with complete information there are formal mechanisms which implement "exactly" the efficient outcome, but this is not the case with incomplete information.

Example 1: There is a population of 100 households who each have the same preferences, $u_i(c_i) = \sqrt{c_i}$. The households differ in the value of their properties: half are of a "low" type with $w_L = 10000$ and the other half are of a "high" type with $w_H = 30000$. Let the probability that a fire burns a given property be 1/100, and be such that exactly one house burns.[22] This allows for easy calculations, and is not much different from the i.i.d. case in terms of incentives and expected utilities. In this case, if other households

[20]When we have more than two farms burning at a time, the argument becomes a bit more complicated, but we can still match up positive and negative terms. The marginal utilities with $m_i - m_i^2/M$ and $w_i - m_i m_j / M$ are replaced respectively by marginal utilities of something like $m_i - m_i(m_i + M_s)/M$ and $w_i - m_i(m_j + M_s)/M$. Again, since $m_i/M \approx 0$, these terms equalize approximately when $w_i = m_i$.

[21]The proof uses the fact that p_s's are equal across s's of the same size. We are not sure how the mechanism performs if there are drastic disparities in the probability of fires across properties. Regardless, *La Crema* could be made to work in such cases by separating properties into relatively homogeneous risk categories operating the mechanism separately over different risk categories, especially as much of the benefits can still be realized with relatively small numbers.

[22]So, $p_1 = \frac{1}{100}$ and $p_k = 0$ for $k \neq 1$, where recall that p_k is the probability of each state where exactly k farms burn.

Table 1: $u_i(c_i) = c_i^{.5}$

	EU_i Autar.	EU_i Truth	EU_i B.R.	Gain B.R.	Best Response
n=2	99.000	99.370	99.420	.050	6000
n=4	99.000	99.452	99.459	.007	7992
n=100	99.000	99.500	99.500	10^{-5}	9925

Table 2: $u_i(c_i) = c_i^{.9}$

	EU_i Autar.	EU_i Truth	EU_i B.R.	Gain B.R.	Best Response
n=2	3941.3	3943.5	3944.1	.6	6000
n=4	3941.3	3944.5	3944.6	.1	8002
n=100	3941.3	3945.2	3945.2	10^{-3}	9925

are reporting truthfully, then a low type's best response is approximately $m_i = 9925$, and the gain in expected utility of announcing 9925 compared to 10000 is approximately 10^{-5} out of an expected utility of approximately 99.5, which is a gain of about only 10^{-5}%. To put this in perspective, not participating leads to an expected utility of 99, and so the overall benefit of participating in *La Crema* is about .5. Thus, the gain of an optimal deviation from truth is very small even compared to the overall benefit from participation ($10^{-5}/.5$). Similar calculations for the high type lead to a best response (to truth by the others) of $m_i = 30077$ and a similar sized gain (on the order of 10^{-5}) compared to truthful announcing.

Table 1 summarizes the results with these parameters for different population sizes.

The results for $u_i(c_i) = c_i^{.9}$ and $u_i(c_i) = c_i^{.1}$ are given in tables 2 and 3 respectively. For more risk averse u_i than the ones we give, the differences between truth and best response are even smaller, and notice that the usual estimated values for the Arrow-Pratt risk aversion parameter[23] are between -1 and -4.

[23] See Szpiro (1986), Barsky et al. (1997) or Chou, Engle and Kane (1992) and references therein.

Table 3: $u_i(c_i) = c_i^{.1}$

	EU_i Autar.	EU_i Truth	EU_i B.R.	Gain B.R.	Best Response
n=2	2.4904	2.5080	2.5085	.0005	6000
n=4	2.4904	2.5090	2.5089	.0001	7982
n=100	2.4904	2.5094	2.5094	–	9925

4.2 Equilibria

While Proposition 4 is somewhat reassuring that truthful reporting of property values can reasonably be expected in the *La Crema* game, it leaves open the possibility that the actual equilibrium could still be quite far from truthful. (Note that generally ε-Nash equilibria need not be near Nash equilibria.) As we now show, however, the Nash equilibria of the *La Crema* game are in fact close to being truthful.

Before we proceed, note that $(0, \ldots, 0)$ is always an equilibrium of the *La Crema* game. We call this the *degenerate* equilibrium. Say that an equilibrium is *non-degenerate* if there is some player i who places probability less than 1 on playing $m_i = 0$. It can be shown that any strategy where $m_i < w_i/2$ is weakly dominated, and so the only equilibria that do not involve weakly dominated strategies must have $m_i \geq w_i/2$ (as is shown in the appendix following equation (8)[24]). In fact, the following propositions show that non-degenerate equilibria exist and have some strong properties.

Proposition 5 *There exists a non-degenerate Nash equilibrium of the La Crema game. Moreover, there exists a strict Nash equilibrium (and thus in pure and undominated strategies) where each player i plays* $m_i \geq \frac{w_i}{2}$ *such that* $\frac{4C^2}{W} \geq |m_i - w_i|$.

The proof of Proposition 5 uses the following Proposition, which establishes that all non-degenerate equilibria involve players playing within certain bounds of w_i.

Proposition 6 *In any non-degenerate Nash equilibrium of the La Crema game, all players only place probability on* m_i *such that*

$$\frac{4C}{W} \geq \left| (m_i - w_i) / w_i \right|.$$

Thus, $\frac{4C^2}{W} \geq |m_i - w_i|$ *and so as* W *becomes large* $|m_i - w_i| \to 0$ *uniformly across* i *for any* m_i *in the support of any sequence of non-degenerate Nash equilibria.*

The proof of proposition 6 follows similar intuition as that behind Proposition 4. We know that the gain from misreporting is small in a large society, and the proof uses the strict concavity of u_i to show that grossly misreporting cannot be a best response: if it involves gross underreporting then there are substantial gains in insurance to be realized by increasing the report, and

[24]In fact, the only time $m_i = 0$ is a best response is if all other players have $m_j = 0$; and as long there is at least one j who places at least some probability on $m_j > 0$, then $m_i = w_i/2$ strictly dominates any lower announcement.

if it involves gross overreporting then there the household is overexposed in their liability and they benefit from decreasing the report.

Propositions 4, 5, and 6 provide a resolution to the seeming conflict between the observation that with heterogeneous societies truthful reporting is not an equilibrium of the *La Crema* game, and the conventional wisdom among the actual participants of the game who think that it is best to report the true value of the property. These previous propositions establish that there exist strict[25] Nash equilibria that are non-degenerate and that any non-degenerate equilibrium of the mechanism is "close" to truthful reporting, and gets closer the bigger the society.

4.3 Approximate Efficiency

While the above results resolve the incentive part of the paradox of the *La Crema* game, the efficiency characteristics are still somewhat puzzling, as with in many cases fully Pareto efficient allocations are not obtainable as an outcome of the game, even under truthful reporting. As it turns out, however, the allocation that results from truthful reporting is close to being efficient in large societies (even with heterogeneous preferences), and thus so are the outcomes associated with non-degenerate equilibria. This is formalized as follows.

Consider a sequence of economies N^h in the *La Crema* game satisfying (A1). Normalize utility functions so that that $u_i(0) = 0$ for each h and $i \in N^h$. Furthermore, suppose that there exists $\underline{a} > 0$ and $\overline{a} > 0$ such that

(A2) $\overline{a} > u_i'(x) > \underline{a}$ for all $x \in [0, 2C]$, h, and $i \in N^h$.

Condition (A2) bounds the derivative of u_i uniformly across i.

Proposition 7 *Consider a sequence of economies N^h in the La Crema game as described above (satisfying (A1) and (A2)). Let the probability of any farm burning be given by $p > 0$ and have this probability be independent across farms.*

(i) If a sequence of risk-sharing allocations $\{x^h\}$ Pareto dominates $\{x^{w,h}\}$ (the allocations associated with truthful reporting in La Crema game), then

$$\frac{\sum_{i \in N^h} Eu_i(x^h) - Eu_i(x^{w,h})}{\sum_{i \in N_h} Eu_i(x^h_w)} \to 0.$$

[25] Such equilibria are also in undominated strategies, and satisfy individual rationality constraints. Note, in fact, that in the *La Crema* game, a player by announcing $m_i = 0$ effectively does not participate, and so any equilibrium must satisfy an interim individual rationality constraint, and here it is satisfied strictly.

(ii) If a sequence of risk-sharing allocations $\{x^h\}$ Pareto dominates the allocations of the La Crema game associated with a non-degenerate Nash equilibrium $\{x^{m,h}\}$, then

$$\frac{\sum_{i\in N_h} Eu_i(x^h) - Eu_i(x^{m,h})}{\sum_{i\in N_h} Eu_i(x^{m,h})} \to 0.$$

The proof of Proposition 7 uses a Law of Large Numbers to tie down the expected property damage to the society. This means that the insurance problem can be approximated by a situation where a given household has a good idea of the cost of insurance and faces only its idiosyncratic risk of loss of property. In such a situation, truthful announcements lead to approximately efficient outcomes, and so non-degenerate equilibria (which are approximately truthful) are also approximately efficient.

5 Conclusions

We have shown that true reporting leads to the unique Pareto efficient outcome of the *La Crema* game, but the corresponding allocation cannot be sustained as an exact equilibrium of this game as long as there is some heterogeneity in household value. However, we have also shown that if the society is large enough, true reporting is "almost" optimal, and that the non-degenerate equilibria of the game lead to outcomes that are close to being Pareto efficient. It is worth remarking that this efficient solution has been attained by a contractual mechanism which is also relatively simple.

Although the framework studied here is one with complete information about the valuations, these results hold even with private information. Truthful reporting is not an equilibrium as long as some agents know that they are likely to have the highest or lowest wealth. But in a large society, deviations will be small, if household believe that their property value will be a relatively small amount of the total announced property value. This robustness with respect to the information structure is important not just because it is more realistic. With complete information there are formal mechanisms which implement "exactly" the efficient outcome, but this is not the case with incomplete information.

Mutual institutions with proportional payment/reimbursement rules are, as we discuss in the introduction, a large part of the insurance business. But they occur in other markets. One is horseracing betting: winning tickets earn back a fraction of total bets in proportion to how much one bets on the winning horse. That is usually referred to as "pari-mutuel"-betting (Gabriel and Marsden 1990, Gulley and Scott 1989). National lottery systems often have this feature as well. This suggests that further exploring the mechanism may be a worthwhile enterprise.

As a final observation, we note that the outcome of the *La Crema* game preserves the relative level of wealth for all households. This contrasts with Young's (1998, p. 132) observation that "the most stable contractual arrangements are those that are efficient, and more or less egalitarian, given the parties' payoff opportunities." An interesting question for future research would be to explain why, of all the possible efficient allocations, the actual mechanism in use results in (something close to) one that preserves the wealth ranking under this class of adverse contingencies.

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Appendix

Proof of Proposition 1: Without loss of generality, assume that $w_n \geq \cdots \geq w_1$. Household i's expected payoff is then:

$$Eu_i(\mathbf{m}) = \sum_{k=1}^{n} Eu_i^{(k)}(\mathbf{m}) + \left(1 - \sum_{k=1}^{n} p_k\right) u_i(w_i)$$

where for all $n \geq k \geq 1$

$$Eu_i^{(k)}(\mathbf{m}) = p_k \left[\sum_{s \in S_i^{(k-1)}} u_i\left(m_i \frac{M_s}{M}\right) + \sum_{s' \in S^{(k)} \setminus S_i^{(k-1)}} u_i\left(w_i - m_i \frac{M - M_{s'}}{M}\right) \right]$$

is the expected utility of household i when k farms burn. Fix some $n \geq k \geq 1$. Direct calculation gives:

$$\frac{\partial Eu_i^{(k)}}{\partial m_i} = p_k \left(1 - \frac{m_i}{M}\right) \Delta_i^{(k)}(m)$$

where

$$\begin{aligned}
\Delta_i^{(k)}(m) &= \sum_{s \in S_i^{(k-1)}} \left(\frac{M_s}{M}\right) u_i'\left(m_i \frac{M_s}{M}\right) - \sum_{s' \in S^{(k)} \setminus S_i^{(k-1)}} \frac{M - M_{s'}}{M} u_i'\left(w_i - m_i \frac{M - M_{s'}}{M}\right) \\
&= \sum_{s \in S_i^{(k-1)}} \sum_{j \in N \setminus s} \frac{m_j}{M} u_i'\left(m_i \frac{M_s}{M}\right) - \sum_{s' \in S^{(k)} \setminus S_i^{(k-1)}} \sum_{j \in s'} \frac{m_j}{M} u_i'\left(w_i - m_i \frac{M - M_{s'}}{M}\right)
\end{aligned}$$

We have $\left|S_i^{(k-1)}\right| = \binom{n-1}{k-1}$ and $\left|S^{(k)} \setminus S_i^{(k-1)}\right| = \binom{n}{k} - \binom{n-1}{k-1} = \binom{n-1}{k}$. Moreover, for all $s \in S_i^{(k-1)}$ and $s' \in S^{(k)} \setminus S_i^{(k-1)}$, $|N \setminus s| = n - k$ and $|s'| = k$. There are thus $(n-k)\binom{n-1}{k-1} = \frac{(n-1)!}{(k-1)!(n-k-1)!}$ elements and $k\binom{n-1}{k} = \frac{(n-1)!}{(k-1)!(n-k-1)!}$ elements respectively on the left-hand side term and on the right-hand side term of $\Delta_i^{(k)}(m)$ that is, an identical number of elements for each sum. We then group these terms two by two in the following way. Let $s \in S_i^{(k-1)}$ and $j \in N \setminus s$. We can write $s = \{i_1 = i, i_2, \ldots, i_k\}$. Let s' be obtained from s by replacing i with j, that is, $s' = \{i_1 = j, i_2, \ldots, i_k\}$. By construction $s \cap s' = \{i_2, \ldots, i_k\}$ implying that $M - M_{s'} = M - M_s - m_i + m_j$. Therefore,

$$\Delta_i^{(k)}(m) = \sum_{s \in S_i^{(k-1)}} \sum_{j \in N \setminus s} \frac{m_j}{M} \left[u_i'\left(m_i \frac{M_s}{M}\right) - u_i'\left(w_i - m_i \frac{M - M_s - m_i + m_j}{M}\right) \right]$$

For all $s \in S$ and $j \in N \setminus s$, let $b_{ii}(s, m) = m_i(M_s/M)$ and $b_{ij}(s, m) = w_i - m_i(M - M_s - m_i + m_j)/M$. Then,

$$\frac{\partial Eu_i}{\partial m_i} = \left(1 - \frac{m_i}{M}\right) \sum_{k=1}^{n} p_k \sum_{s \in S_i^{(k-1)}} \sum_{j \in N \setminus s} \frac{m_j}{M} [u_i'(b_{ii}(s, m)) - u_i'(b_{ij}(s, m))] \tag{1}$$

In particular, when $\mathbf{m} = \mathbf{w} = (w_1, \ldots, w_n)$, and letting $W = \sum_{i \in N} w_i$ and, for all $s \in S$, $W_s = W - \sum_{i \in s} w_i$ (the remaining wealth after firms in s have burnt) we get:

$$\left.\frac{\partial Eu_i}{\partial m_i}\right|_{\mathbf{m}=\mathbf{w}}$$
$$= \left(1 - \frac{w_i}{W}\right) \sum_{k=1}^{n} p_k \sum_{s \in S_i^{(k-1)}} \sum_{j \in N \setminus s} \frac{w_j}{W} \left[u_i'\left(w_i \frac{W_s}{W}\right) - u_i'\left(w_i \frac{W_s - w_i + w_j}{W}\right)\right] \tag{2}$$

Suppose that for some $i, j \in N$, $w_i \neq w_j$. Then clearly $w_n > w_1$, implying that $\left.\frac{\partial Eu_1}{\partial m_1}\right|_{\mathbf{m}=\mathbf{w}} < 0$ and $\left.\frac{\partial Eu_n}{\partial m_n}\right|_{\mathbf{m}=\mathbf{w}} > 0$. In words, the poorest (resp. the richest) household has strict incentives to underreport (resp. overreport) and $\mathbf{w} = (w_1, \ldots, w_n)$ is not a Nash equilibrium of the *La Crema* game. If on the contrary $w_1 = w_n = w$ then for all $i \in N$, $w_i = w$ and $\left.\frac{\partial Eu_i}{\partial m_i}\right|_{\mathbf{m}=\mathbf{w}} = 0$ implying that $\mathbf{w} = (w_1, \ldots, w_n)$ is a Nash equilibrium of the *La Crema* game. ▮

Proof of Remark 1: We proceed in five steps.

1. Let us show first that (m_1, m_2) with $m_i \neq 0$ and $m_i \neq 2C$ for all $i \in \{1, 2\}$ cannot be an equilibrium. If $m' = (m_1, m_2)$ were an equilibria we would have:

$$\begin{cases} \left.\frac{\partial Eu_1}{\partial m_1}\right|_{\mathbf{m}=m'} = 0 \\ \left.\frac{\partial Eu_2}{\partial m_2}\right|_{\mathbf{m}=m'} = 0 \end{cases} \Leftrightarrow \begin{cases} \frac{m_1 m_2}{m_1+m_2} = w_1 - \frac{m_1 m_2}{m_1+m_2} \\ \frac{m_1 m_2}{m_1+m_2} = w_2 - \frac{m_1 m_2}{m_1+m_2} \end{cases} \Leftrightarrow \begin{cases} w_1 = 2\frac{m_1 m_2}{m_1+m_2} \\ w_2 = 2\frac{m_1 m_2}{m_1+m_2} \end{cases}$$

which is impossible.

2. The profile $(m_1', 0)$, with $m_1' \neq 0$, cannot be an equilibrium as

$$\left.\frac{\partial Eu_2}{\partial m_2}\right|_{\mathbf{m}=(m_1',0)} = p_1[u_2'(0) - u_2'(w_2)] > 0.$$

Similarly, $(0, m_2')$, with $m_2' \neq 0$, cannot be an equilibrium.

3. The profile $(2C, m_2)$ is not a Nash equilibrium. To see this, notice that the best response to $2C$ is $m_2' = \frac{2w_2C}{4C-w_2}$, since $\left.\frac{\partial Eu_2}{\partial m_2}\right|_{\mathbf{m}=(2C,m_2')} = 0$, and $m_2 = 0$, $m_2 = 2C$ produce lower payoffs than m_2' against $2C$. However, the best response to $\frac{2w_2C}{4C-w_2}$ is not $2C$, but rather $m_1 = (w_1\frac{2w_2C}{4C-w_2})/(\frac{4w_2C}{4C-w_2} - w_1)$ (which by assumption is smaller than $2C$, as one can directly verify that this expression is less than $2C$ whenever $w_1 < w_2$).

4. The profile $(\frac{2w_1C}{4C-w_1}, 2C)$, is a Nash equilibrium. First, the unique best response to $2C$ is $m_1' = \frac{2w_1C}{4C-w_1}$. This also implies that $(\frac{2w_1C}{4C-w_1}, 2C)$, with

$m_1 \neq \frac{2w_1 C}{4C - w_1}$ is not an equilibrium. Then notice that the only point m_2' at which $\left.\frac{\partial Eu_2}{\partial m_2}\right|_{\mathbf{m}=\ (m_1', m_2')} = 0$ is $m_2' = (w_2 \frac{2w_1 C}{4C-w_1})/(\frac{4w_1 C}{4C-w_1} - w_2)$, and by assumption $m_2' < 0$ (noting that the denominator is less than 0 if and only if $w_1 \left(1 + \frac{w_2}{4C}\right) < w_2$). Also, $\left.\frac{\partial Eu_2}{\partial m_2}\right|_{\mathbf{m}=\ (m_1', 0)} = p_1[u_2'(0) - u_2'(w_2)] > 0$, which added to the fact that $m_2' < 0$ and continuity implies that $\left.\frac{\partial Eu_2}{\partial m_2}\right|_{\mathbf{m}=\ (m_1', C)} > 0$.

5. The only remaining case is $m = (0, 0)$. This is trivially an equilibrium. The payoff to any player i in this case is that of autarky, independently of the choice of m_i. ∎

Proof of Proposition 2: Let a consumption vector $\mathbf{c} \in \mathbb{R}^{2^n}$, $MRS_i^{r,s}(\mathbf{c}) = \frac{p_r \partial u_i/\partial c_r}{p_s \partial u_i/\partial c_s}$ denotes the marginal rate of substitution of player $i \in N$ between two states $r, s \in S$ with respective probabilities p_r and p_s. Pareto efficient allocation are characterized by equal marginal rates of substitution across all agents in N for all states in S. In particular, given a message vector $\mathbf{m} \in [0, 2C]^n$ and $r, s \in S^{(1)} \backslash \left\{S_i^{(0)} \cup S_j^{(0)}\right\}$, $MRS_i^{r,s}(\mathbf{c}(\mathbf{m})) = MRS_j^{r,s}(\mathbf{c}(\mathbf{m}))$ is equivalent to

$$\frac{u_i'\left(w_i - m_i \frac{M-M_r}{M}\right)}{u_i'\left(w_i - m_i \frac{M-M_s}{M}\right)} = \frac{u_j'\left(w_j - m_j \frac{M-M_r}{M}\right)}{u_j'\left(w_j - m_j \frac{M-M_s}{M}\right)}.$$

With identical CRRA utility functions we get

$$\frac{w_i - m_i \frac{M-M_r}{M}}{w_i - m_i \frac{M-M_s}{M}} = \frac{w_j - m_j \frac{M-M_r}{M}}{w_j - m_j \frac{M-M_s}{M}} \Leftrightarrow (M_s - M_r)(m_i w_j - m_j w_i) = 0.$$

Therefore, either there exists some $\lambda \in \mathbb{R}$ such that $m_k = \lambda w_k$, $\forall k \in N$, or $m_k = m_l = m$, $\forall k, l \in N$. Now, let $s = S_i^{(0)}$ and $r \in S^{(1)} \backslash \left\{S_i^{(0)} \cup S_j^{(0)}\right\}$. Then, $MRS_i^{r,s}(\mathbf{c}(\mathbf{m})) = MRS_j^{r,s}(\mathbf{c}(\mathbf{m}))$ is equivalent to

$$\frac{w_i - m_i \frac{M-M_r}{M}}{m_i - m_i \frac{m_i}{M}} = \frac{w_j - m_j \frac{M-M_r}{M}}{w_j - m_j \frac{m_i}{M}}.$$

If $m_k = m$, $\forall k \in N$ this expression is equivalent to $(w_j - \frac{m}{n})/(m - \frac{m}{n}) = 1$, $\forall i, j \in N$ which is incompatible with $w_i \neq w_j$ for some $i, j \in N$. We are thus left with $m_k = \lambda w_k$, $\forall k \in N$ for some $\lambda \in \mathbb{R}$. let $s \in S_i^{(0)}$ and $r \in S^{(0)}$. Then, $MRS_i^{r,s}(\mathbf{c}(\mathbf{m})) = MRS_j^{r,s}(\mathbf{c}(\mathbf{m}))$ is equivalent to

$$\frac{m_i - m_i \frac{m_i}{M}}{w_i} = \frac{w_j - m_j \frac{m_i}{M}}{w_j} \Leftrightarrow \lambda = 1.$$

Moreover, it is easy to check that all other marginal rates of substitution are equalized across agents when $m_i = w_i$, $\forall i \in N$. ∎

Proof of Remark 2: Let the Walrasian price for a unit of consumption in state s be q_s. The efficient allocation of *La Crema* leads to consumption of $w_i \frac{W_s}{W}$ for agent i in state s. Assume, for a contradiction, that the efficient allocation is a Walrasian equilibrium. The budget constraint is given by:

$$\sum_{s \in S} q_s \frac{w_i W_s}{W} = \sum_{s \notin S_i} q_s w_i$$

and dividing on both sides of the equation by w_i, we obtain

$$\sum_{s \in S} q_s \frac{W_s}{W} = \sum_{s \notin S_i} q_s \tag{3}$$

Optimality requires that the marginal relation of substitution between any two states r, s is equal to the ratio of consumption prices between these states. Let us normalize the price of consumption in the state where no farm burn ($r = \{0\}$) to 1. This implies that

$$\frac{p_s u_i'\left(w_i \frac{w_S}{w}\right)}{p_0 u_i'(w_i)} = \frac{p_s \left(w_i \frac{W_s}{W}\right)^{\alpha-1}}{p_0 w_i^{\alpha-1}} = \frac{p_s}{p_{\{0\}}} \left(\frac{W_s}{W}\right)^{\alpha-1} = q_s$$

substituting the price in (3) it follows that for each i:

$$\sum_{s \in S} \frac{p_s}{p_0} \left(\frac{W_s}{W}\right)^{\alpha} = \sum_{s \notin S_i} \frac{p_s}{p_0} \left(\frac{W_s}{W}\right)^{\alpha-1}$$

Eliminating the p_0 we have

$$\sum_{s \in S} p_s \left(\frac{W_s}{W}\right)^{\alpha} = \sum_{s \notin S_i} p_s \left(\frac{W_s}{W}\right)^{\alpha-1}$$

Let S^{-i} denote the states that would exist if i were not in the economy. So, S has twice as many states as S^{-i}. For $s' \in S^{-i}$, let $W_{s'}$ be the wealth in state s' if i were not in the economy. Keep W as the total wealth including i and p as the probability that a farm burns. We rewrite the above expression as

$$\sum_{s' \in S^{-i}} p_{s'} \left[(1-p)\left(\frac{W_{s'} + w_i}{W}\right)^{\alpha} + p\left(\frac{W_{s'}}{W}\right)^{\alpha}\right]$$
$$= \sum_{s' \in S^{-i}} p_{s'}(1-p)\left(\frac{W_{s'} + w_i}{W}\right)^{\alpha-1}$$

Rearranging terms we get that

$$\sum_{s' \in S^{-i}} p_{s'} \left[(1-p)\left(\frac{W_{s'} + w_i}{W}\right)^{\alpha}\left(1 - \left(\frac{W_{s'} + w_i}{W}\right)^{-1}\right) + p\left(\frac{W_{s'}}{W}\right)^{\alpha}\right] = 0 \tag{4}$$

must hold for each i. Now, let $S^{-j,k}$ denote the set of states where neither j nor k are in the economy. Rewriting (4) when $i=j$ we get that

$$\sum_{s''\in S^{-j,k}} p_{s''}\left[(1-p)^2\left(\frac{W_{s''}+w_j+w_k}{W}\right)^{\alpha}\left(1-\left(\frac{W_{s''}+w_j+w_k}{W}\right)^{-1}\right)\right.$$
$$+(1-p)p\left(\frac{W_{s''}+w_j}{W}\right)^{\alpha}\left(1-\left(\frac{W_{s''}+w_j}{W}\right)^{-1}\right)$$
$$\left.+p(1-p)\left(\frac{W_{s''}+w_k}{W}\right)^{\alpha}+p^2\left(\frac{W_{s''}}{W}\right)^{\alpha}\right]=0 \tag{5}$$

Similarly, from k's perspective we get

$$\sum_{s''\in S^{-j,k}} p_{s''}\left[(1-p)^2\left(\frac{W_{s''}+w_j+w_k}{W}\right)^{\alpha}\left(1-\left(\frac{W_{s''}+w_j+w_k}{W}\right)^{-1}\right)\right.$$
$$+(1-p)p\left(\frac{W_{s''}+w_k}{W}\right)^{\alpha}\left(1-\left(\frac{W_{s''}+w_k}{W}\right)^{-1}\right)$$
$$\left.+p(1-p)\left(\frac{W_{s''}+w_j}{W}\right)^{\alpha}+p^2\left(\frac{W_{s''}}{W}\right)^{\alpha}\right]=0 \tag{6}$$

Subtracting (6) from (5) we get

$$\sum_{s''\in S^{-j,k}} p_{s''}(1-p)p\left[\left(\frac{W_{s''}+w_k}{W}\right)^{\alpha-1}-\left(\frac{W_{s''}+w_j}{W}\right)^{\alpha-1}\right]=0$$

But this cannot hold if $w_k<w_j$ or if $w_k>w_j$, which is a contradiction. ▮

Proof of Proposition 3: Let us first consider $n\geq 3$. Given a message vector $m\in[0,2C]^n$ and $r\in S^{(1)}\backslash\left\{S_i^{(0)}\cup S_j^{(0)}\right\}$, $s\in S^{(0)}$, $MRS_i^{r,s}(c(m))=MRS_j^{r,s}(c(m))$ is equivalent with identical CARA utility functions to

$$w_i-m_i\frac{M-M_r}{M}-w_i=w_j-m_j\frac{M-M_r}{M}-w_j\Leftrightarrow m_i=m_j$$

Now, let $r\in S_i^{(0)}, s\in S^{(0)}$. Then $MRS_i^{r,s}(\mathbf{c}(\mathbf{m}))=MRS_j^{r,s}(\mathbf{c}(\mathbf{m}))$ is equivalent to

$$m_i-m_i\frac{m_i}{M}-w_i=w_j-m_j\frac{m_i}{M}-w_j$$

Since $m_i=m_j$, this is equivalent to

$$m_i-m_i\frac{m_i}{M}-w_i=-m_i\frac{m_i}{M}\Leftrightarrow m_i=w_i$$

Similarly we can also show that $m_j = w_j$, which is a contradiction with $m_i = m_j$ and $w_i \neq w_j$. Now let $n = 2$. Then, for $r \in S_1^{(0)}, s \in S^{(0)}, MRS_1^{1,0}(\mathbf{c}(\mathbf{m})) = MRS_2^{1,0}(\mathbf{c}(\mathbf{m}))$ is equivalent to

$$m_1 - m_1 \frac{m_1}{m_1 + m_2} - w_1 = -m_1 \frac{m_2}{m_1 + m_2} \Leftrightarrow w_1 = m_1 \frac{m_2}{m_1 + m_2}$$

Similarly we can show that $w_2 = m_2 \frac{m_1}{m_1+m_2}$, which is a contradiction with $w_1 \neq w_2$. ∎

Proof of Remark 3: Let a message vector $m \in [0, 2C]^n$ and $r, s \in S^{(1)} \backslash \left\{ S_i^{(0)} \cup S_j^{(0)} \right\}$, $MRS_1^{r,s}(\mathbf{c}(\mathbf{m})) = MRS_i^{r,s}(c(m))$ is equivalent with CRRA utility functions to

$$1 = \frac{w_i - m_i \frac{M - M_r}{M}}{w_i - m_i \frac{M - M_s}{M}} \Leftrightarrow M_r = M_s$$

Now, let $s \in S_i^{(0)}, r \in S^{(1)} \backslash \left\{ S_i^{(0)} \cup S_j^{(0)} \right\}$. Then $MRS_1^{s,r}(\mathbf{c}(\mathbf{m})) = MRS_i^{s,r}(\mathbf{c}(\mathbf{m}))$ is

$$1 = \frac{m_i - m_i \frac{m_i}{M}}{w_i - m_i \frac{m_s}{M}} \Leftrightarrow w_i = m_i$$

The previous two equalities imply that

$$w_i = w_j$$

which is a contradiction. ∎

Proof of Proposition 4: Fix h. We bound $\frac{\partial E u_i(w)}{\partial m_i}$ by an expression that is decreasing in n^h.

From (2) we know that

$$\left. \frac{\partial E u_i}{\partial m_i} \right|_{\mathbf{m}=\mathbf{w}}$$

$$= \left(1 - \frac{w_i}{W}\right) \sum_{k=1}^{n} p_k \sum_{s \in S_i^{(k-1)}} \sum_{j \in N \backslash s} \frac{w_j}{W} \left[u_i'\left(w_i \frac{W_s}{W}\right) - u_i'\left(w_i \frac{W_s - w_i + w_j}{W}\right) \right]$$

This implies that

$$\left| \left. \frac{\partial E u_i}{\partial m_i} \right|_{\mathbf{m}=\mathbf{w}} \right| < \max_{s \in S_i^{(k-1)}, j \notin s} \left| u_i'\left(w_i \frac{W_s}{W}\right) - u_i'\left(w_i \frac{W_s - w_i + w_j}{W}\right) \right| \quad (7)$$

Note that

$$\left| w_i \frac{W_s}{W} - w_i \frac{W_s - w_i + w_j}{W} \right| < C \frac{C - c}{n^h c}$$

Then by (7) and (A1), for any $\mu > 0$ we can find H_μ such that for any $h > H_\mu$,

$$\left| \left. \frac{\partial E u_i}{\partial m_i} \right|_{\mathbf{m}=\mathbf{w}} \right| < \mu$$

for all $i \in N^h$. Finally, given any ε choose μ such that $\mu = \frac{\varepsilon}{2C}$. Given the strict concavity of u_i, it follows that the maximal gain from a report of some m_i instead of w_i is $2C|\partial E u_i / \partial m_i| < \varepsilon$ for all $i \in N^h$ where $h > H_\mu$. This establishes the proposition. ▮

Proof of Proposition 6: Let σ be a non-degenerate Nash equilibrium of the *La Crema* game. Consider i and a strategy profile σ_{-i} that does not place probability 1 on all players $j \neq i$ playing 0. From (1) we know that

$$\frac{\partial E u_i(m)}{\partial m_i} = \left(1 - \frac{m_i}{M}\right) \sum_{k=1}^{n} p_k \sum_{s \in S_i^{(k-1)}} \sum_{j \in N \setminus s} \frac{m_j}{M} [u_i' (b_{ii}(s,m)) - u_i' (b_{ij}(s,m))]$$

where b_{ii} and b_{ij} are as defined in the proof of Proposition 1. Consider any strategy profile m_i, σ_{-i}.

$$\frac{\partial E u_i (m_i, \sigma_{-i})}{\partial m_i} =$$

$$\int \left[\left(1 - \frac{m_i}{M}\right) \sum_{k=1}^{n} p_k \sum_{s \in S_i^{(k-1)}} \sum_{j \in N \setminus s} \frac{m_j}{M} [u_i' (b_{ii}(s,m)) - u_i' (b_{ij}(s,m))] \right] d\sigma_{-i}(m_{-i}) \tag{8}$$

Note that we can reverse the order of integration with respect to m_{-i} and derivation with respect to m_i (i.e., differentiate inside the integral in getting the above expression) because the function $\partial E u_i (m_i, \sigma_{-i}) / \partial m_i$ of m_i is bounded on player i's strategy set $[0, 2C]$ for all σ_{-i}. Note that (8) implies that any strategy with $m_i < w_i/2$ is weakly dominated. This follows from noting that $b_{ii}(s,m) < m_i$ and $b_{ij}(s,m) \geq w_i - m_i$ for any s and m_{-i}, and so given the strict concavity of u_i the expression is strictly positive regardless of s and m_{-i}, provided that $m_i < w_i/2$.

Let (s^*, m^*) minimize $u_i'(b_{ii}(s^*, m^*)) - u_i'(b_{ij}(s^*, m^*))$ over the support of m_i, σ_{-i}. (8) and the concavity of u_i also imply that

$$\frac{\partial E u_i (m_i, \sigma_{-i})}{\partial m_i} \geq$$

$$\int \left[\left(1 - \frac{m_i}{M}\right) \sum_{k=1}^{n} p_k \sum_{s \in S_i^{(k-1)}} \sum_{j \in N \setminus s} \frac{m_j}{M} [u_i'(b_{ii}(s^*, m^*)) - u_i'(b_{ij}(s^*, m^*))] \right] d\sigma_{-i}(m_{-i})$$

Thus,

$$\frac{\partial E u_i (m_i, \sigma_{-i})}{\partial m_i} \geq$$

$$[u_i'(b_{ii}(s^*,m^*)) - u_i'(b_{ij}(s^*,m^*))] \int \left(1 - \frac{m_i}{M}\right) \sum_{k=1}^{n} p_k \sum_{s \in S_i^{(k-1)}} \sum_{j \in N \setminus s} \frac{m_j}{M} d\sigma_{-i}(m_{-i}) \tag{9}$$

Given that σ_{-i} that does not place probability 1 on all players $j \neq i$ playing 0, the integral on the right hand side of (9) is strictly positive. Then it follows from (9) that $0 \geq \partial Eu_i(m_i, \sigma_{-i})/\partial m_i$ implies that

$$0 \geq u_i'(b_{ii}(s^*,m^*)) - u_i'(b_{ij}(s^*,m^*))$$

Thus, given (A1), $0 \geq \partial Eu_i(m_i, \sigma_{-i})/\partial m_i$ implies that $b_{ii}(s^*,m^*)) \geq b_{ij}(s^*,m^*)$, which can be rewritten as

$$m_i \geq \frac{w_i}{1 + \frac{m_i - m_j^*}{M^*}} \tag{10}$$

Noting that $1 \geq \frac{m_i - m_j^*}{M^*} \geq -1$, it follows from (10) that if σ_{-i} does not place probability 1 on all players $j \neq i$ playing 0, then a best reply by i must have support only on $m_i \geq w_i/2$. This then implies that if σ is a mixed strategy equilibrium that does not place probability 1 on $(0, \ldots, 0)$, it must be that the support of each σ_j is a subset of $[w_j/2, 2C]$. This implies that $\frac{4C}{W} \geq \frac{m_i - m_j^*}{M^*}$, and so from (10) it follows that if σ is a mixed strategy equilibrium that does not place probability 1 on $(0, \ldots, 0)$, it must be that for each i and any m_i that is a best response to σ_{-i}

$$m_i \geq w_i/(1 + \frac{4C}{W}) \tag{11}$$

Let (s^{**}, m^{**}) maximize $u_i'(b_{ii}(s^{**},m^{**})) - u_i'(b_{ij}(s^{**},m^{**}))$ over the support of m_i, σ_{-i}. If σ_{-i} has the support of each σ_j as a subset of $[w_j/2, 2C]$ then (8) and the concavity of u_i imply that

$$[u_i'(b_{ii}(s^{**},m^{**})) - u_i'(b_{ij}(s^{**},m^{**}))] \int \left(1 - \frac{m_i}{M}\right) \sum_{k=1}^{n} p_k \sum_{s \in S_i^{(k-1)}} \sum_{j \in N \setminus s} \frac{m_j}{M} d\sigma_{-i}(m_{-i})$$

$$\geq \frac{\partial Eu_i\left(m_i, \sigma_{-i}\right)}{\partial m_i}$$

Thus, $\partial Eu_i(m_i, \sigma_{-i})/\partial m_i \geq 0$ implies

$$\frac{w_i}{1 + \frac{m_i - m_j^{**}}{M^{**}}} \geq m_i \tag{12}$$

Since $\frac{4C}{W} \geq \frac{m_i - m_j^{**}}{M^{**}}$ it follows that

$$w_i/(1 - 4C/W) \geq m_i \tag{13}$$

(11) and (13) establish the proposition. ▮

Proof of Proposition 5: The fact that any pure strategy non-degenerate equilibrium only involves play of $m_i \geq w/2$ such that $\frac{4C^2}{W} \geq |m_i - w_i|$ follows directly from the proof of Proposition 6. Let us show that there exists such an equilibrium and that it is a strict equilibrium. We do this by showing that to any best response of m_{-i} such that $m_j \in [w_j/2, 2C]$ there is a unique best response (which then must be in $[w_i/2, 2C]$ by Proposition 6) that varies continuously in m_{-i}. The result then follows from Kakutani's Theorem.

From the proof of Proposition 6 it follows that $\partial Eu_i(m)/\partial m_i$ is continuous in m_i and m_{-i}, and that $\partial Eu_i(m)/\partial m_i > 0$ if $m_i < w_i/2$, and $\partial Eu_i(m_i)/\partial m_i < 0$ if $m_i > \frac{w_i}{1-\frac{4C}{W}}$. Thus, there exists a point $m_i \in [w_i/2, \frac{w_i}{1-\frac{4C}{W}}]$ where $\partial Eu_i(m_i)/\partial m_i = 0$. We show that at any such point $\partial^2 Eu_i(m)/\partial m_i^2 > 0$. This implies that there are no local minima which in turn implies that there is a unique such point. Direct calculation gives

$$\begin{cases} \frac{\partial b_{ii}}{\partial m_i} = \left(1 - \frac{m_i}{M}\right)\left(\frac{M_s}{M}\right) \\ \frac{\partial b_{ij}}{\partial m_i} = -\frac{(M-m_i)}{M}\left(\frac{M-M_s-m_i+m_j}{M}\right), \forall j \neq i \\ \frac{\partial}{\partial m_i}\left[\frac{m_j}{M}\left(1 - \frac{m_i}{M}\right)\right] = -\frac{2m_j}{M^2}\left(1 - \frac{m_i}{M}\right) \end{cases}$$

leading to

$$\frac{\partial^2 Eu_i(m)}{\partial m_i^2} =$$

$$\underbrace{\left(1 - \frac{m_i}{M}\right) \sum_{k=1}^{n} p_k \sum_{s \in S_i^{(k-1)}} \sum_{j \in N \setminus s} \frac{m_j}{M} u_i''(b_{ii}) \left(1 - \frac{m_i}{M}\right)\left(\frac{M_s}{M}\right)}_{<0}$$

$$+ \underbrace{\left(1 - \frac{m_i}{M}\right) \sum_{k=1}^{n} p_k \sum_{s \in S_i^{(k-1)}} \sum_{j \in N \setminus s} \frac{m_j}{M} u_i''(b_{ij}) \left(\frac{M - m_i}{M}\right)\left(\frac{M - M_s - m_i + m_j}{M}\right)}_{<0}$$

$$- \left(1 - \frac{m_i}{M}\right) \sum_{k=1}^{n} p_k \sum_{s \in S_i^{(k-1)}} \sum_{j \in N \setminus s} \frac{2m_j}{M^2} [\, u_i'(b_{ii}) - u_i'(b_{ij})]$$

This second expression is $-\frac{2}{M}\partial Eu_i(m)/\partial m_i$ and so the whole expression is negative whenever $\partial Eu_i(m)/\partial m_i \geq 0$. This concludes the proof. ▮

Proof of Proposition 7: We prove (i), as then (ii) follows in a straightforward way from Proposition 5 (and (A2)). Let $\overline{W}^h = E^h[W_s^h]$. The Weak

Law of Large Numbers[26] implies that

$$\text{Prob}^h \left[\left| \frac{W_s^h - \overline{W}^h}{W^h} \right| \geq \varepsilon \right] \to 0 \tag{14}$$

for any $\varepsilon > 0$. It follows from (14), the continuity and bounds on u_i that for any $\varepsilon > 0$ there exists H such that

$$\text{Prob}^h \left[\left| u_i \left(w_i \frac{W_s^h}{W^h} \right) - u_i \left(w_i \frac{\overline{W}^h}{W^h} \right) \right| \geq \varepsilon \right] < \varepsilon \tag{15}$$

for all $i \in N^h$ and any $h > H$. Let x^h Pareto dominate $x^{w,h}$. Suppose to the contrary of the Proposition that there exists $\delta > 0$ such that

$$\frac{\sum_{i \in N^h} Eu_i(x^h) - Eu_i(x^{w,h})}{\sum_{i \in N_h} Eu_i(x^{w,h})} > \delta$$

for infinitely many h. Let $\overline{x}^h = (E[x_1^h], \ldots, E[x_{n^h}^h])$ be the expected value of x^h. Then by the concavity of u_i,

$$\frac{\sum_{i \in N^h} u_i(\overline{x}^h) - Eu_i(x^{w,h})}{\sum_{i \in N_h} Eu_i(x^{w,h})} > \delta \tag{16}$$

for infinitely many h. Given (A2), it follows from (16) and the fact that x^h Pareto dominates $x^{w,h}$ that for each such h we can find some $\gamma > 0$ and vector $\widehat{x}^h$ such that $\sum_{i \in N^h} \widehat{x}_i^h = \sum_{i \in N^h} \overline{x}_i^h$ and

$$u_i(\widehat{x}_i^h) - Eu_i(x_i^{w,h}) > \gamma \tag{17}$$

for all $i \in N^h$. Then from (15) it follows that

$$u_i(\widehat{x}_i^h) - u_i \left(w_i \frac{\overline{W}^h}{W^h} \right) > \gamma$$

for all $i \in N^h$, for infinitely many h. However, as both $\widehat{x}^h$ and $w_i \frac{\overline{W}^h}{W^h}$ sum to $\overline{W}^h$, this is a contradiction. ∎

[26]We apply a version covering sequences of heterogeneous but independent random variables (e.g., see Billingsley Theorem 6.2 in the 1979 edition). Note here that $\sigma^h / W^h \to 0$ where σ^h is the standard deviation of W_s^h. This follows since $C\sqrt{n^h p(1-p)} \geq \sigma^h$ and $W^h \geq nc$.